Time Function Waves, Interference Integrals and Abstract Field Theory

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Abstract. Observing time domain properties of a network (delays, locations, time functions at locations, velocities) the main behavior of a net can be observed. We will call such networks "Interference Networks" (IN). By contrast to Neural Nets (NN) all signals on wires of IN have distributed (inherent) delays. (The term "interference" means an universal superimposition or interaction of (mostly non-periodic, spiking and delayed) time functions f(t - T)).

The paper addresses questions of a better understanding of pictures of thought, sound maps or movement maps [4–15] in nerve systems in the same way, as it addresses technical applications (Acoustics, Radar, Sonar, lens systems, feedback controls, GPS). It gives an overview about the high potential of signal interference in nerves and in circuit theory. An-alyzing the spherical flow of *time functions*, we find them to be *waves*. IN create an *abstract wave theory without materialistic background*. This background gives a huge possibility to synchronize knowledge of different scientific fields. It has potential to combine parts of wave optics, neural nets, acoustics, filter theory, control theory, electron-physics and neuroscience under one abstract, physical roof. The IN-approach creates a high potential for education of students if introduced as basic lecture.

"The question, how the nervous system creates representations of its environment has fascinated philosophers and scientists since mankind began to reflect on its own nature." Wolf Singer, 1993

1 History of Interference Networks (IN)

End of the 80th mankind had knowledge about a lot of artificial "neuronal" nets, lots of works were done about learning nets, oscillatory nets or spatio-temporal maps, about holography or coherence, for example see [1,2,16–22]. The output map of such networks represented in nerve like parametrisation the input map - if. By coincidence in September 1992 I found a little problem: If we suppose geometrically small impulses that flow very slow through such nets, it is not possible to get non-mirrored output presentations (interference integrals) that correspond to input maps. Like optical lens systems such nets can only produce *mirrored* projections. 1993 I spent a half year to find any mirrored map in neurocomputing literature, but I could not found anything.

Source:

Heinz, G.: Time Function Waves, Interference Integrals and Abstract Field Theory. Paper rejected from Second International Work-Conference on the Interplay between Natural and Artificial Computation (Iwinac 2007), Murcia (Spain), June 18-21, 2007 By the way: the inspiration was reasoned by thinking about a simple multichannel Radar-system for cars. I found, that *continuously running time* in such systems can only produce *mirrored maps* with additional hints like *axial-near sharpness* known from optics. To get high-quality images, we need time reversal algorithms¹.



Fig. 1. Classification of interference networks

Limited velocity of nerve impulses supposed (μ m/s ... m/s) [11, 13, 15] any millisecond impulse becomes a geometrical wave length in the range between nanometer and millimeter (v = ds/dt): The geometrical length of a pulse can be very short in comparison to the size of a neuron.

Interference nets can be seen like cross-roads: the probability that cars (pulses) coming from different directions (dendrites) crashes on the crossing is as higher, as smaller the distances between the cars or as longer the cars or as slower (!) they are (ds = v dt). Static signals (EPSP...) at logic circuits (soma) are comparable to infinite long trains crashing statically at the crossing. In nerves with pulse/pause ratios of 1:10 to 1:10.000 the "crash probability" for excitement is very small. What to do? First we find, that static signal processing (pattern nets) is inadequate for data processing. Second, we have to look for "crash" places! We have to follow a single impulse or signal over the whole network, hoping it meets

¹ The paper has nothing to do with electric or magnetic or acoustic fields: we examine waves and wave field integrals in time domain only, *without of any materialistic background*.

his double(s) at certain places - we have to look for (discrete) interference locations of signals, for discrete *pulse wave interferences*.

Introducing the approach we find, that *nerve networks* (in opposite to neural nets) map the input pattern only *mirrored* to the output! In September 1992 this idea was like a shock: It was not possible to find any scientific publication about a mirroring property in neuro-computing literature. The problem becomes as bigger, as more such wave analogies lead to optical projections. Like a interference circuit in nerve dimensions a simple, optical lens system mirrors the image. The next shock was, that I could not find much about elementary wave conditions for optical projections, looking for abstract wave-conditions.

So the idea was born to investigate the field of "discrete wave interference on distributed, wired nets". Can a physical approach to neural nets (later called "interference nets") create a connection between wave physics (optical, acoustical) and neuro-computing?

2 Character of Interference Networks (IN)

By contrast to "neural" networks (NN) the wires of IN need distributed delays. Wires carry velocities, delays and spatial information. The time functions flow on the wires with constant or variable speed, with or without attenuation. IN demand simulations in time domain. Choice of a rough time or space grid or improper use of time function parameters destroy the wave properties of an interesting IN immediately. Spatial arrangements of bundles of wires, studied in [15], showed the influence of geometrical changes to wave fronts on the bundle: "space codes behavior". It is necessary to define the space arrangement of each wire. In meaning of interference we use the term "discrete wave" instead of "signal" to manifest this property. We find following properties of IN:

- Physical nets, continuous in space and in time
- Distributed delays on all wires (wires are not electrical nodes)
- No information flow without delay (!)
- Wires carry time functions f(t-T)
- Spatial wire definition is necessary f(x, y, z)
- Classical neuron definitions are possible (integrate & fire etc.)
- Generated pulses are carried on different wires and meet again

Thinking this way, we find time functions to be (abstract) waves. Interference nets create an abstract and non-materialistic wave theory on inhomogeneous $nets^2$.

² Remark about wave calculus: We demonstrate only simple systems. Complex multimeshed systems develop for example partial, "eating" waves, compare [4], Fig.7. They show much complex behavior.

3 Non-Orthogonal Vector/Matrix Notation

We consider a time function $f(t - \tau)$ in digital notation as vector of samples f_j ; $F = f(t - \tau) = [f_1, f_2, ..., f_n]$. Addition of time function vectors is analog to matrix conventions, $F_1 + F_2$ but *multiplication* of time functions is defined *non-orthogonal* sample by sample, see [15].

$$F_1 * F_2 = [f_{11}, f_{12}, \dots f_{1n}] * [f_{21}, f_{22}, \dots f_{2n}]$$
(1)

$$= [f_{11}f_{21}, f_{12}f_{22}, \dots f_{1n}f_{2n}]$$
(2)

Remark 1. Note, that classic convolution (with a core $f(t) * g(\tau - t)$) is a source of confusion in interference networks. A convolution has nothing to do with wave interference. It is necessary, if you bring a wave in interference with a standing wave function, for example to give a soma a specific filter task if the wave runs about this place.

A further source of confusion concerns the use of *Fourier-Transformation* or other integral transformations for wave theories of every kind. The 0...360° limitation of complex number theory brings confusion into (pulse-like) interference systems. We lose a lot of information doing calculations in frequency domain, if we use distances that are larger the smallest wavelength in the system (usual case).

4 Time Function as Wave

Calculating each pixel of a n-dimensional field, we suppose (if not otherwise noted) a *delay proportional to distance* between any source point and the actual pixel. We suppose a limited velocity for all signals. This simplest method generates *moving waves*, if we observe subsequent time steps in a movie.

Example 1. The source point of f(t) has the coordinates $P(x_0, y_0, z_0)$. At a pixel $P_1(x_1, y_1, z_1)$ the function may appear delayed by τ_1 , the time function is there $f_{P1}(t) = f(t-\tau_1)$. At a different pixel $P_2(x_2, y_2, z_2)$ it appears delayed by τ_2 with time function $f_{P2}(t) = f(t-\tau_2)$ and so on.

If time functions of different source locations move over such a field, they interfere at some points. Integration over long times, this points of interference are highlighted (-> interference integral), [15].

Basically this technology was used since 1993 to construct the Acoustic Camera, nominated for different awards, beyond a nomination for the German Future Award 2005.

Remark 2. Note, that time function waves have no physical background. We need no materialistic substrate. We only need to know the named proportions: time functions, velocities, locations. This point of view seems to generate a new, abstract field theory, suitable for nerve nets or technical things, if we can get only time domain measures of any subject.

5 Foundation: Self- and Cross Interference

If events (pulses) occurred by the same origin meet again, we have to observe two, very different cases. The case if a single impulse *i* meet again his deviates *i* (sorry for the abstract terms), we call self-interference (*Selbstinterferenz*, case a). If we use a sequence of source pulses (a pulse series $i, i + 1, i + 2 \dots$), additional we have to investigate the correspondence of predecessors and followers. We call the interference of impulses with a different origin cross-interference (*Fremdinterferenz*, Fig.2, cases b).

While high channel numbers in delay-adjusted systems reproduce the selfinterferences (images), low channel numbers underline the cross interference locations (frequency maps), [12].



Fig. 2. Self-interference a) and cross-interference between pulses b)

Investigating such wave networks we find capabilities for informational tasks, like *temporal to temporal* coding (*bursts*), *spatial to spatial* coding (*projections*), *temporal to spatial* coding (*frequency* maps) or *spatial to temporal* coding (creation of *behavior*), [15].

6 Interference Integrals

If we integrate for a long time over a multi-channel wave field, points of wave interference become highlighted. We call the result in optics a photograph, in common speaking an interference integral or in acoustics an acoustic image. Also a frequency map is a interference integral, small channel numbers enhance here the cross interference parts.

Supposed, any neuron receives signals (waves) from n different sources, Fig.3. The (projective) sum of interferences g(t) of n delaying time functions f_k is at time t and location $P(x_0, y_0, z_0)$

$$g(t) = \frac{1}{n} \sum_{k=1}^{n} f_k(t - \tau_k), \ k = 1...n,$$
(3)

with delay vector (mask)

$$M = (\tau_1, \tau_1, ..., \tau_n).$$
(4)

The *interference integral* of n by t_k delayed time functions in a time interval T is a value. By analogy to electrical systems for example the *effective value* is

$$y_{eff} = \sqrt{\lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} [\frac{1}{n} \sum_{k=1}^{n} f_k(t - \tau_k)]^2 dt.}$$
(5)



Fig. 3. Time function g(t) of point P summing four sources $f_k(t)$

The equation produces a delay vector M [12]. If pre-delayed by a different $M' \neq M$ g(t) get more and more noise, as more M' differs from M. Maximum interference occurs in P if functions $f_k(t)$ appear pre-delayed with the negative vector -M of P (velocity can be slow in neural space).

Opposite case: If a neuron produces an excitement at any location P it burns its delay vector M as address into the resulting time functions (Fig.3). Any spherical shift of P is followed by a different delay vector. That means, the *delay vector represents the location of* P. Looking back into the time functions of Fig.3, we find M looks like a mask. So the *interference reconstruction* can be realised using a so called mask algorithm [3]. To get any interference time function g(t) we have to shift the delay mask M of g(t) over the channels $f_k(t)$, adding vertically sample by sample over the holes. Using $f(t + \tau)$ for all pixels this is the main idea for the calculation of acoustic images and films. Doing this, we get a non-mirrored reconstruction of the scene, without of sharpness and over-conditioning problems.



Fig. 4. Expansion of waves in 3D-space. A different P produces a different M

6.1 Projection Equation

Independent, if we consider optics or acoustics or neural nets we find a well known but not named law: locations of interference (the maximized interference integral) are there, where all partial waves from the excitement point come into coherence again. This *point of self interference* has the additional condition, that delay sums on all paths are equal. The sum of delay vector elements of the generating field M_G , the delay vector of the transmitting lines M_T and the delay vector of the detecting field M_D have to be equal. [1] symbolizes the unit vector [12].

$$M_G + M_T + M_D = \tau[1] \tag{6}$$

(self interference condition)

By analogy we construct different cross interference conditions, see [15].

6.2 **Projection and Reconstruction**

For technical purposes we differ between *projection* (optics, beam forming) and (computational, non-causal) *reconstruction*. Using $f(t + \tau)$ we get the so called *reconstruction* (Acoustic Camera), using $f(t-\tau)$ we get the *projection*. While the reconstruction delivers a 1:1 image, the projection produces *mirrored* interference integral images [12] with axial sharpness problems know from optics, see Fig.5b). In case of perfect reconstruction the τ in last equation will be zero.

6.3 Conditioning

If pulses of the same origin meet n-times again, the question of conditioning appears. Using a d-dimensional sphere, we need d + 1 channels (waves) to mark precise the self interference location, n = d+1. Using more channels we get overconditioned projections (for example optical lens projections). With a smaller channel number the projection is under-conditioned, it moves. For example we get hyperbolic excitement curves for the case of two channels on a two dimensional surface [10] (n = d : under-conditioned). For real space the dimension is limited to d = 3. Nerve system can increase the dimension (and following the channel number) only using inhomogeneous spaces by velocity-variation (axonal/dendritic diameter changes) and spatial convolution (cortex) [?, 4–15].

6.4 Address Volume

Nerve velocities and pulse length can be very small compared to the dendritic and axonal size of a neuron [11,13,15]. Any geometrical pulse width λ determines the sharpness maximum of a pulse projection on a core (soma), it is defined by the pulse peak time t_{peak} and velocity v; $\lambda = t_{peak}v$. If a neuron must be addressable independent of neighbors, the average distance between neurons is limited to λ .

Example 2. With 10 μ m wave length, velocity 10 mm/s, pulse width 1 ms we can address a maximum grid of 10x10x10 μ m³ per liter, these are 10¹² neurons. Interesting: as slower the velocity (as slower the animal), as smaller is the geometric pulse width and as higher is the capacity, however.

7 Temporal to Temporal Coding - Bursts

By analogy to FIR- and IIR-digital filters Fig.5 shows a neuron-like interference circuit, that produces time functions (bursts) b) or that works like a time-function (burst) detector c). All wires might have distributed delays [10]. Using a b-type neuron as generator and a c-type neuron with the negative delay vector -M as detector, such neuron pairs can communicate independent via special bursts on a single line. I called the principle *data-addressing*. If a neural pair has mask-pairs, that are not inverse, the neurons will not communicate.



Fig. 5. Basic functions of a neuron or a neural group a) Circuit structure, b) Burst generation with low bias, c) Burst detection with high bias

We can find this effect in case of two neurons with the same spatial structure. If they have identical delay vectors, they avoid uncontrolled feedback between them. So connected, nearest neurons with identical structure can not communicate! We call this *dynamical neighborhood inhibition*. In case, the wavelength is much higher the size of a neuron, or pulses come overlapped in interference, a neuron has the ability to generate floating values, necessary for bias control or for velocity controls via glia-potential [12]. Burst generation, burst detection, data-addressing, neighborhood inhibition and control level generation we will find as *dynamical, elementary functions of neurones* [8, 10].

8 Spatial to Spatial Coding - Self Interference Projections

A certain excitement (G) in Fig.6 produces a highest interference integral at the interference location (E). This is the place, where all partial waves meet again in self-interference, the delays are equal on all pathways $\tau_1 = \tau_2 = \tau_3$. To find locations of interference numerically, the region of interest can be considered as very dense mashed - like a continuous, free wave surface, b). Each co-ordinate in the generator field maps mirroring on a certain co-ordinate in the receiving field.



Fig. 6. Spatial self interference a) projection principle, b) example (over-conditioned reconstruction top and projection bottom)

In [12] some projection-variants were published. Changing the velocity between generator and detector field the projection size *zooms*, the projected image becomes greater or smaller. Changing the delay on any pathway (channel) between generator and detector the projected image *moves* to a different place, *well conditioned projections* with n = d + 1 supposed (for example n = 3 channels for d = 2 surfaces) [12].

8.1 Composition and Decomposition

A special sort of projections, called scene composition or decomposition, changes the dimension of an interference projection. For example a 3D-scene (channel number n = 4) P1234 can correspond to different synchronized 1D-scenes (n = 2) P12, P23, P34, P41 or to corresponding 2-dimensional scenes (n = 3)P123, P234, P341, P412 and so on. Because of cross-interference noise reasoned by small pulse distances, this is a way to compose projections into high dimensions, see [7, 12, 15]. It allows any interference net (nerve system) a much higher data throughput without reaching the cross-interference limitations of Fig.9.

9 Temporal to Spatial Coding - Cross interference as frequency map

If a "split wave" (time function) with the same origin meets again, we obtain a *cross interference map*, see Fig.7. The geometrical distances of the cross interference maxima appear as function of the geometrical arrangement and as function of the time function parameters (pulse frequency or the pause between pulses - refractory period).



Fig. 7. Frequency map as spatial code of a frequency. a) Two channel circuit example; b) result for two channels and c) three channels. Simulation: PSI-Tools, gh 1996

Function: While a self interference of wave i with wave i (written: [i, i]) produces the centered emission only (-> projection), any cross-interference of events (pulses) i with i-1, i+1, i-2, i+2 ... produces a map with emission distances proportional to pulse pause (refractory) distance, a frequency map.

10 Spatial to Temporal Coding

Any nerve fiber delay is proportional to length. A code generator in form of Fig.5b produces an output code (time function), that is carried by the intrinsic delays of the structure. So each spherical arrangement produces a certain arrangement of time-functions - space codes behavior or structure defines the function [15].



Fig. 8. Trajectory examination. If an event (for example a pulse) runs along the trajectory, a specific set of delays will detect it

11 Mixed Coding Forms

11.1 Trajectory Examination of a Moving Source

Looking on interference locations, we get a natural way to detect trajectories of moving sources. Supposed we have some in succession firing cells creating a trajectory in form of a moving figure. Neurons on the trajectory (Fig.8) fire consecutively. Interference maximum occurs in P with delay vector $M = (\tau_1, \tau_2, ..., \tau_n)$ for $\tau_n = \tau_{n-1} - \tau$, with $\tau = ds/v$, if ds is a distance and v is the velocity of movement [7, 15]. If a local field potential (glia) controls the velocity or the delays τ_n , different velocities can be observed by variation of field potential.

11.2 Fire Density, Holographic Projection and Pain

Lashley [20] analyzed the location of memorization with trained rats. Independent, which part of the brain he removed, the rats could remember partially a learned behavior, a way through a labyrinth. Remembering, that each impulse is followed practically all the time by a further impulse, self-interference emissions in form of a G are surrounded always by cross-interference figures, Fig.9a. What a surprise, they look similar? The delay between pulses defines the crossinterference distance, the distance between the "G" figures. Any memorization is



Fig. 9. Mirroring three channel projection of a "G". a) Cross interference residua around a self interference figure; b), c), d) cross interference overflow produced by increasing fire rates

closely coupled to, what Bohm and Pribram [2, 16, 22] called, holographic content, holographic measure etc.. So Lashley had *theoretically* no chance to find clear locations of memorized contents - what a genius concept of nature!

But what would happen if we reduce the pause between pulses? The cross interferences comes nearer and nearer, Fig.9b...d [10]. At a certain point the cross interferences overlay the self-interference locations: the projection disappears. If we remember, that the fire rate of sensory neurons increase dramatically in case of injury, we can imagine a possible mechanism of pain.

11.3 Topomorphic Overlayed Projections

In our imagination it is possible, to overlay images or impressions. Can we find any theoretical evidence for such behavior? To test this, we overlay two channel data streams. The generating fields 'g' and 'h' have identical channel numbers. They project into the same fields, Fig.10, [12] by overlaying (add, append) the channel data streams. Both generator images combine in the receiving fields. If channel source points are moved within the detector fields, the projections become distorted. But the projections maintain in topomorphic relation. It is not possible to separate them again.

12 Technical Applications

To demonstrate capabilities of IN I tried to develop a technology to produce acoustic images and films, using a back-propagating algorithm basing on inverse waves with non-causal delays f(t+T) [3], [9]. The acoustic camera technology become worldwide successful and accepted, it is used in most car development centers in the world (www.acoustic-camera.com). The camera was nominated for different awards, in 2005 we were nominated for the German Future Award. Lots of journalists reported about the technology, see www.gfai.de/~heinz/publications/presse.



Fig. 10. Topomorphic relations between time functions of two sources 'g' and 'h', interfering on different fields



 ${\bf Fig.\,11.}$ FIR-filter as a simple, specific interference network

Thinking about interference networks, we find the global positioning system (GPS), digital filters, or control loops can be seen as such circuits. More: Any synchronized system acts as a simplest IN, for example a data latch of a computer or the mixer in a radio receiver. A digital filter (Fig.11) for example can be seen as a discrete, very simple interference network variant of Fig.5.

For all IN-systems the *function depends on the arrival time of external events* (inputs). For example: An elevator moves to different floors, if persons on the floors push the buttons at different times. This might be surprising, it means that *interference networks bring abstract wave fields into analog and digital circuit theory* at one side, and allow simulations of complexest connected and delayed circuits (nerve nets) on the other. Compare with www.gfai.de/~heinz/publications and find introductions at www.gfai.de/~heinz/historic.

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