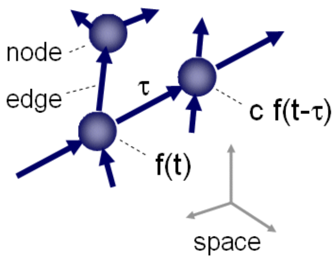
Non-Recursive Interference Calculi

A Mathematical Calculus Immanent in Nervous Activity

Gerd Karl Heinz

Abstract Interference networks (IN) and interference systems have a comparable mathematical-physical background, reaching from photonic wave interference in optics over signal interference in digital filters (FIR, IIR), wave interference in Radar- or Sonar- devices to ionic pulse interference in nerve nets. Special properties of IN are short wavelength, relative timing and non-locality of function. Behind concepts of cybernetics and informatics, we find in interference integrals a hidden functional principle for nerve nets, the non-locality of function. The paper highlights integration methods in non-recursive IN. It reflects on simulation movies on the web [9].



Any process, that uses as input a set of time functions (acoustic, ionic, electric or photonic) and delivers as output 'images' or 'maps' denotes an interference process. The paper observes interference networks without feedback between nodes (non-recursive IN); the edges carry mono-directional time-functions.
The idea of modeling such processes bases on the conjunction (addition/multiplication) of spatial delayed, *synchrotopic* time functions at

Fig. 1: Non- recursive interference network (IN). Any time function *f(t)* arrives a next node delayed by τ and damped by *d* in a form $d * f(t-\tau)$.

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locations or nodes, and the integration of interference values. Synchrotopy combines in Greek the words *syn-* for together, *chronos* for time and *topos* for location [7].

In difference to solutions of differential equations at single nodes (wave equation) the interest of IN is to understand the function of networks.

In interference systems, the type of operation, the type and parameters of time-function, the type of non-linear exponation or integration and the current channel number 'interfere'.

Various works showed properties of interference integrals ('wave images'): In acoustic imaging [4], for nerve nets [8], for high-speed microcontrollers [7]. Simplicity of the element-by-element integration process on the one hand; and complexity to find possible parameterizations on the other constrained earlier publication.

We suggest a simplest model. It is a non-recursive conjunction without feedback; delays shift time- and space-functions without feedback.

Reflecting McCulloch/Pitts paper [12] "A logic calculus immanent in nervous activity" interference systems or networks have to contain more then a logic calculus; physical aspects (delaying wave space) and additional mathematical aspects (integration – this paper) play an important rule too.

Analyzing interference nets, we find different philosophical aspects compared to computer science and neuroscience. While state machines (Boolean nets, also McCulloch/Pitts) use concepts, that *associates a 'function' or 'signal' physically to an output of a cell or gate*, wave spaces and interference nets show a different functional organization and mapping. Thinking about Radar maps or acoustic images, the *relativity of timing of signals defines the function at a certain place; a function does not necessarily associate with the output of a gate or cell.* Function has no locality. Any gate or cell output can carry very different signals in sequence. Non-local functionality needs something like a dense gate-ground (the retina for example). This seems to be the main aspects to clarify retarding progress: Every philosophic assumption about nerve nets up to now assumes a computer-like binding of function to an axonal output like "PORTB3 - LED on/off".

> "A profound revolution lurks in our basic concept of how the information-bearing elements of the nervous system communicate." T.H. Bullock, [1]

Inspecting a homogeneous photonic wave field on a photo plate, we can shift the plate before exposure – the resulting (wave-) image after exposure is independent of the shift and of the exact location of the net parts.

If we change the position or location of some pins of a bus in our PC, the system would not work further. For the airplane-helicopter crash [7], the absolute place of the crash plays no rule. If machines start and fly with identical relative timing, motion, wind, height and direction, they could meet again.

In IN the type of conjunction (multiplicative or additive), the time function type (positive or symmetrical), the way to calculate the detection (integration or averaging), the time function properties (single pulses or sinoidal forms) and the channel numbers influence the properties of integrals (maps, images) [7].

For example, multiplication of symmetrical time functions works best for two, but not for more channels. Additive conjunction is just satisfying for many channels but works not satisfying for two.

Zero Delay Prohibited

Any incoming time-function $f(t)$ leaves the edge delayed by τ and damped by a factor d (mostly $d=1$)

$$f(t) \rightarrow d * f(t-\tau) \tag{1}$$

If not otherwise noted, the edge delays the transmitted signal. Each node remarks a location with a spatial position or location within the network, defining the delay properties of the net. Information needs time to bridge spaces. Infinite fast transmission (without a physical delay) is impossible. In opposite to electric schematic drawings, where edges mark equipotential connections (electric nodes), interference nets have only edges with (distributed or fixed) delays. Comparable to electrics, the equipotential assumption of nodes is valid only within a node. Every edge has a delay. By analogy, non-local calculations are impossible. For example, the convolution integral needs delayed inputs. The IN-representation splits into different nodes and external delays.

Space- and Time Function, Wave Function

Public wave function definition (Wikipedia) is different to the model proposed here. Things are much simpler. We suggest only a dense meshed network and any time function that can freely expand into all directions.

Time function and space function show different views of the same thing:

- A time function $g(t-\tau)$ is valid for a single point (oscillogram).
- A space function $\psi = f(vt-r)$ is valid for a single time step (ocean wave).

Space and time function join by analogy with velocity $v = r/\tau$, delay τ, radius r. If a time function is leaving one-dimensional space, it becomes a wave. Main characteristic of a wave is that all points of a certain peak have the same delay to the source location.

Only the calculation of radius r in space function $\psi = f(vt-r)$ has to change to show one- or higher dimensional waves. In $r = x-x_0$ the index x_0 denotes the locations of the source point of wave, while x denotes the actual location. For Euclidian space for example the radius r is the well known Euclidian distance *one dimensional space* $$r = x - x_0 \tag{2}$$

two dimensional space $$r = \sqrt{(x - x_0)^2 + (y - y_0)^2} \tag{3}$$

three dimensional space $$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \tag{4}$$

If we follow a single peak of the time function through any space, we find it as a wave [8]. Consequently, a wave characterizes nothing more and nothing less, then a space-function in spherical dimension higher one.

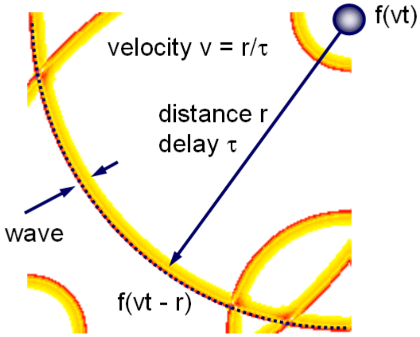

Fig. 2: Example of a 2-dim. Euclidian wave function $\Psi = f(r,t) = f(vt\text{-}r)$. With $v = r/\tau$ it relates to time function waves of type $g(t\text{-}\tau)$.

If we inspect any system with the property to delay signals flowing through, we can regard the signals to be wave-like. The form of expansion of waves depends on the spherical measure or norm [7]. Euclidian space geometry produces round waves. Manhattan geometry shows squared waves. Inhomogeneous space geometry of nerve nets shows broken, distorted or discontinuous waves [10].

Independent of the measure, all nodes (pixel etc.) under the wave $\psi = f(vt\text{-}r)$ have a common property: they have the same delay distance (delay) to the source point of the wave $f(t)$.

To finish the wave concept, any single node can overlay different waves coming from different origins [4]. We find this kind of wave propagation in various types of substrates for different carriers of waves; we find it in air or water (electric-, photonic-, acoustic waves), on integrated circuits (electric waves) or in nerve nets (ionic waves). By contrast, Andrew Packard's squid experiments on nerve nets [13] had shown that nerve systems do not mesh all nodes necessarily to all sources. Inspecting his records it seems, we find yellow and red waves interacting independent of black waves. We talk here about different layers.

Suggesting a dense meshed network with intrinsic delays between nodes, if we run the parameter *vt*, the waves come in motion. Running it forward, the wave runs forward with wave peak in front and wave tail back. If we run the time parameter backward, the wave runs backward showing the source point. Using a negative delaying substrate [4], we get waves going inward with tail outside.

Compare with examples and movies on the web page [9]. Under the movies lays the Scilab [14] source code.

Race Circuits

If the geometric wave length $\lambda = vT$ comes in the range of the size of a circuit or arrangement, we talk about 'race circuits'. Interference nets are race circuits.

Functions of race circuits need any kind of integration processes. If we use a paper sheet instead of the photo plate, after exposure no image remains. The photo image needs integration over lots of waves to form the interference integral – the image – within the light sensing material of the photo plate.

In case of electronic race circuits, race conditions occur in the range of femtoseconds for each state. Radar, electric interferometers, reflectometers or femtosecond-meters use the idea in several ways. Each pixel of any optical image shows properties of interference integration. Fast integrated circuits show more and more the problem of race conditions, because each single wire delays the carried signal relative to the length. Last not least acoustic camera or Sonar uses the interference of sound waves.

Delays and Waves

The main important property of interference systems is the restriction that every signal needs time to reach a destination at a different location [5].

For the exposure of a photo plate, a small deformation of the delaying wave space by a minimal mismatch of orthogonal direction to the axis destroys the image.

The delay assumption transforms a simple time function into a *wave function* [7]. The interference network becomes a *wave space*. We talk about homogeneous or inhomogeneous wave spaces.

While nerves need inhomogeneous wave spaces, acoustic, optic or electric net models use mostly a homogeneous, Euclidian geometry. Dependent of application we find different distance measures for delay, for example Euclidian, Manhattan or chaotic (nerves) [7].

Node Structure of Non-Recursive Nets

Independent of network type the signal processing subdivides into the processes of input weighting, conjunction and transfer.

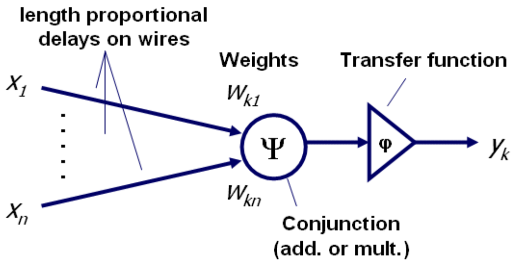

Fig. 3: Net node structure

Separation of interference network nodes into the three parts weighting, conjunction and integration allows descriptions of different processes and types of nets: ionic, electric, photonic or acoustic nets. Modeling the dendrite tree of a nerve, the edge between weighting and conjunction can also have a delay. The conjunction- operator Ψ symbolizes a substitute for possible conjunctions of time functions in interfering systems of different kind like Ψ = {Π, Σ, Λ (AND), V (OR), max, min, exp}. A general model of node function [6, 7], that fits some cases, is

$$y_k(x, y, z, t) = \varphi(\Psi_{j=1}^{n} w_{kj} x_j(t - \tau_{kj})) \tag{5}$$

Transfer functions can be fuzzy sets, threshold functions or integral functions.

Interaction of Time Function and Processing

It might be trivial to underline, that the type of time function (bipolar-symmetric or monopolar-asymmetric/positive) has influence on possible calculations.

Multiplication of bipolar signals on more then two channels is impossible. Using many channels and multiplication, any negative part swaps the interference integral. Hence, computation of many unipolar signals is restricted to addition (acoustic cameras) or to multiplication of monopolar signals (nerve system).

By opposite, addition produces integral values also, if only a single time function has a value and all other deliver zero. For acoustic imaging this means that every still location in a resulting map gets a residual interference integral value.

Using two channels, multiplication has interesting properties. It is in common use for signal processing (modulation, frequency doubling etc.) and for any kind of computer algebra (logical AND).

Last not least we know from Fuzzy Sets, the kind of conjunction (addition – multiplication) influences the probability, that any stochastic computation shows the maximum (addition, OR) or the minimum (multiplication, AND).

Interference Detection: Integral or Average?

Signal processing and electronics use a wider terminology for 'integration' compared to integrals in mathematics. In electronics, we call any accumulation of values an integration process. To register and count very fast and small spikes, it is possible to observe integrals. Using integrative solutions we differ between two general types of interference detection, that is

- Interference integration and
- Interferential averaging.

While acoustic cameras use an averaging process to reconstruct maps with dB-values, any photo film or plate shows an integral behavior. As longer is the exposure time, as brighter becomes the film.

Anti-Parallel Addition of Waves - d'Alembert Waves

A first public solution of the wave equation shows d'Alembert's formula. However, the solution restricts interference to a one-dimensional principle with anti-parallel data flow. It is dedicated to analyze concurrent edges and violates restrictions for non-recursive interference nets.

A solution of the one-dimensional wave equation

$$\frac{1}{c^2}\frac{\delta^2 u}{\delta t^2} = \frac{\delta^2 u}{\delta x^2} \tag{6}$$

is d'Alembert's formula, for example in the form

$$u(t,x) = u_1(ct + x) + u_2(ct - x) \tag{7}$$

Two time-functions $u(x,t) = u_1(x,t) + u_2(x,t)$ interfere here on a one-dimensional sphere (wire, axon etc.). Neglecting the details, d'Alembert's formula allows also this solution. The public solution $u(x,t) = u_1(x+ct) + u_2(x-ct)$ shows one forward $u_1(x+ct)$ and one backward traveling wave $u_2(x-ct)$ [9]. The wave trail runs there in front of the wave. We use the form with trails at the end of waves.

Any interference integral appears only, if we introduce a non-linearity that enforces the place of wave meeting. Integration of added time functions shows always a *residual interference* value at locations, where one of the time functions is zero. That means, the integral value at any location tends to increase independent of other properties.

Multiplication of Waves

Multiplication of positive time functions removes the integral at all locations, where one of the time functions is zero. The approach

$$u(t,x) = u_1(ct+x) * u_2(ct-x) \tag{8}$$

has special potential for demonstrational purposes [9] and in signal processing, for example for modulation, demodulation or coding (AM, FM, GPS). Using single waves, the interference integral appears at the place of wave meeting, see animations. If one of the contra-directional flowing time functions comes later, the meeting place of time functions shifts to a location in direction of the delayed function.

Pixel Calculus and Time-Parallel Data Flow

To overcome the restrictions of one-dimensional contra- directional waves, we apply a node-convention. In opposite to anti-parallel calculation it simplifies the task. The node abstraction suggests, that spherical arriving waves show a time-parallel data flow. It allows the calculation of waves in *n*-dimensional networks.

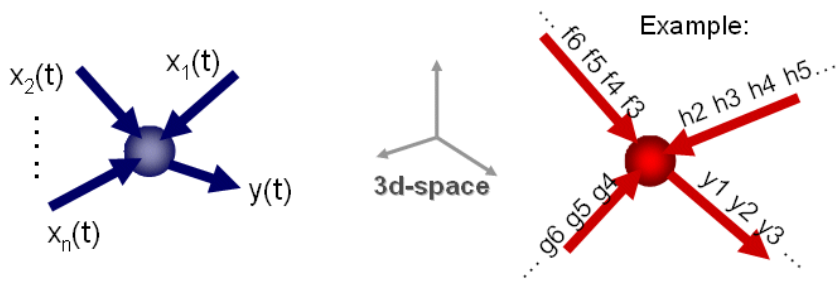

Fig. 4: Time-Parallel Data Flow (Pixel Calculus). The example demonstrates the time parallelism: For numerical addition we can write $y1 = f3+g4+h2$, $y2 = f4+g5+h3$, $y3 = f5+g6+h4$ etc. In Scilab we write only $y = f+g+h$, if y,f,g,h are equal sized vectors.

At each node of the IN, the multi-directivity in space corresponds now to a parallelism of data flow in time dimension.

If time functions are lists, they relate to element-by-element computation. Scilab [14] writes element-by-element multiplication of vectors with dot-star, for example $c = a\ .*\ b$.

Examples for Interference Systems

Open Integration - Photography, Ultra Short Time Measurement

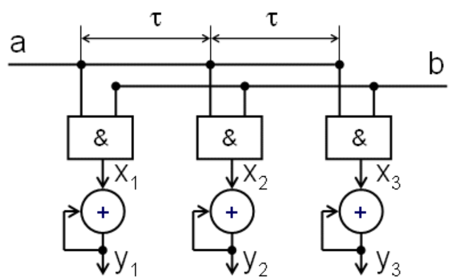

Fig. 5: Example of a Boolean race logic with additional delay difference 2τ (!) and integrator (compare with data set in Tab.1)

Supposing, two spiking time-functions (*a, b*) in the range {0…1} with zero-level {0} and information level {1} meet in a way comparable to Fig.1. The gate inputs are a_i and b_i, the gate delivers the output $x_i = a_i \& b_i$ (Boolean AND-operation), i is the gate index.

Resulting x_i spikes apply in electric interference applications generally to short, to register them. To get a possibility for registration, we have to integrate over x_i to get a higher *accumulation* y_i. The slope adds each result of x_i to y_i (integration) for each location or net node (we have three here):

$$x_i = a_i \,\&\, b_i \longrightarrow y_i = y_i + x_i \quad \text{(integration)} \tag{9}$$

The value of y_i shows the interference integral level. It shows, how many Ones meet at the gate.

To see the effect of any change in position, the τ in Fig.1 delays the related time function *a* or *b* by two time steps, Tab.1. We suppose infinite time function sequences. Any vector *a, b, x, y* is a small part of a infinite long sequence. The normal position of gates may be in the middle (x_2, y_2). Additional delays mark the left or right positions (x_1, y_1) and (x_3, y_3).

Left and right positions can symbolize two cases:

- one of the time functions *a* or *b* is delayed (here by two steps)

- the middle gate moves the position to left or right (with additional delay)

Both cases are equivalent. Each 'hit' of Ones in (*a=1* & *b=1*) produces a result $x = 1$ and increases the respective accumulator y by one.

Using a time function a and a time function b that shifts on the time scale (left/right) by two steps ($\tau \sim n = 2$) relative, the effect of delay shift on the interference integral can be studied by example, Tab.1 and Fig.1.

In case of maximum interference of Ones between a and b (middle position, x_2) the accumulation is maximal ($y_2 = 6$). A variation of delay between inputs a and b by two steps shows, that the interference integral value y varies (y_1, y_3). The value of the accumulator y_i marks the number of hits.

Table 1: Interference integrals of two time functions. Example for an 'open' integration process for logic gates (data set for Fig.4):

Direction of time flow →

```
a(t)    =[ 0 0 0 1 0 0 1 1 1 0 0 0 0 0 1 1 0 0 0 ]
b(t-2)  =[ 0 0 1 1 0 0 0 0 1 1 1 0 0 0 0 1 1 1 0 ]
x1      =[ 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 ]
y1      =[ 0 0 0 1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 3 ] = 3

a(t)    =[ 0 0 0 1 0 0 1 1 1 0 0 0 0 0 1 1 0 0 0 ]
b(t)    =[ 1 1 0 0 0 0 1 1 1 0 0 0 0 1 1 1 0 0 0 ]
x2      =[ 0 0 0 0 0 0 1 1 1 0 0 0 0 1 1 1 0 0 0 ]
y2      =[ 0 0 0 0 0 0 1 2 3 3 3 3 3 4 5 6 6 6 6 ] = 6

a(t-2)  =[ 0 0 0 0 1 0 0 1 1 1 0 0 0 0 0 1 1 0 0 ]
b(t)    =[ 1 0 0 0 0 1 1 1 0 0 0 0 1 1 1 0 0 0 0 ]
x3      =[ 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 ]
y3      =[ 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 ] = 1
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Photonic Interference and a Photonic Paradox

Inspecting light waves and their interference integrals (images) we find a paradox. The addition of many time functions at a detecting field (retina, photo plate, screen) produces in theory no interference integral, if we compare photonic waves with symmetric, sinoidal waves, see the movie in [9] (Sinoidales Integral bei Addition, #*sinadd*). If a single wave has at a certain location an integral value of zero, also the addition of many waves delivers zero. To get an image, we have to suppose any nonlinearity. The detecting field produces the solution: it has to transform the linear overlay by a nonlinear operation using the squaring terms 'energy' or 'intensity'.

Light waves interfere on a photo film, producing the image as interference integral. As longer is the exposure time, as brighter is the exposure. Film exposure appears proportional to shutter opening time.

Let *x(t)* be the sum of all light waves reaching the target place, the resulting time function *y(t)* of the film is the integral over time. Suggesting the time function *x(t)* of light consists of *periodic, symmetric waves* we know from DC-current, the *interference integral is always zero* [9]. Positive integral parts delete the negative,

$$y(t) = \int_{-\infty}^{\infty} x(t)\,dt = 0 \tag{10}$$

To get any exposure different to zero, the observed time function *x(t)* cannot be *periodic and area-symmetric* to time axis, see the animation films [9]. The interference integral *disappears for symmetric time functions*.

If the time function integral *y(t)* has to be different to zero, we have to suppose a non-linear rectifier (film, CCD, eye) within the chain, squaring the time function

$$Y(t) = \int_{-\infty}^{\infty} (x(t))^2\,dt \neq 0\,. \qquad \text{(energy, intensity)} \tag{11}$$

If the effect of the positive part of wave to any photo sensor or film is identical to the effect of a even high negative part, we talk about energy or intensity.

Overexposure of Photo Plates and Integral Divergence for Open Integration

As longer is the integration time, as more increases the value of any interference integral for a photo film. The value depends directly from the length of measuring interval. The interference integral accumulates to a *steady growing* time-function of the pixel or sensor. We will call it an ‘*open*’ *integration process*. The accumulation grows to infinite for infinite long time or sequences. If not zero, the resulting integral is divergent in most practical cases

$$f(t) \to \infty \quad for \quad t \to \infty\,. \tag{12}$$

If the Ones are interfering Planck photons, we have to stop the integration process after a certain time to avoid overexposure.

Investigation of photography brought the *shutter* to stop overexposure of interference integration process on a photo plate. Open integration stops after the exposure time. The accumulator resets and the next image can start by opening again the shutter.

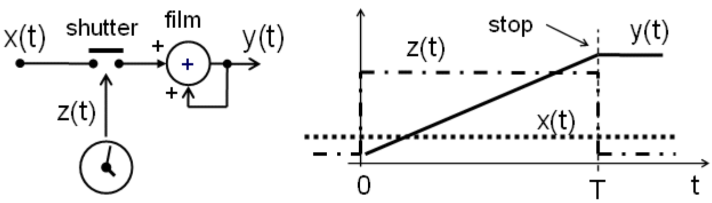

Fig. 6: Equivalent circuit for open loop integration of photographic films

Integration with Loops – CCD, Nerves

To limit the level of exposure, we can stop the exposure at the exposure maximum of the sensor or we can downscale the sensitivity relative with the maximum of the integral, for example using a moving average algorithm. Processes use sometimes a servo loop; we call them 'loop integration'.

Stop-Loop Integration – CCD Camera Chipsets

Automatic exposure systems in CCD-camera chip sets stop the light exposure, if the first of the sensor pixels of the camera reaches the integration maximum. If y(t) exceeds *max*, the comparator sends a one. If not, it sends a zero value to the output. The shutter closes the input for all pixels, if maximum (max) exceeds. The accumulator value of each pixel represents now the light intensity.

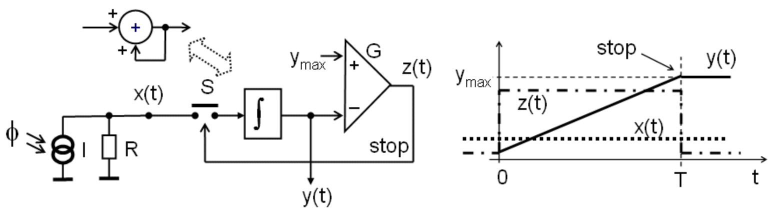

Fig. 7: Equivalent circuit for servo loop integration of CCD camera chip sets: photo-cell signal *x(t)* integrated signal y(t), comparator G. Shutter opens when *y(t)* exceeds the maximum value y_{max}.

After reading the pixel values by the computer, the next image exposure starts with a reset of the accumulator value to *y(t) = 0* (not drawn).

Pulse Frequency Integration (with loop) – Eyes, Nerves

Translating the open integration method to our eyes means, as longer we look on an object, as more lightness our eyes would get. We know this is not true. We can look hours on an object without overexposure of retina. Nerves and eyes use a different integration method.

While video cameras (CCD) conserve the accumulator until the computer has read it, by opposite nerve cells destroy the accumulator value just in the minute, the accumulator is full. It seems to make no sense.

But integration time reflects now the input current. A high input current (high optical flux) brings a short integration interval, a low current a long integration interval. In the minute, the accumulator value *y(t)* at comparator *comp* is greater the maximum value y_{max}, the comparator output changes to high, reseting the accumulator.

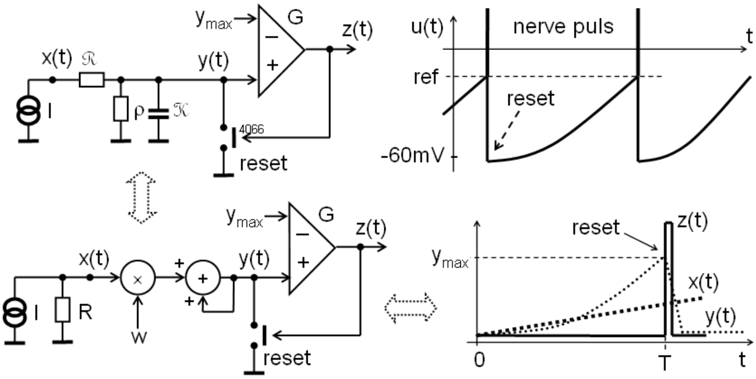

Fig. 8: Integrate-and-fire-neuron [2] as integrator in IN. First row: Lapique's R-C network of 1907 [11] enhanced by a comparator and a reset-loop. Right: Pulse principle of nerve. Second raw: Equivalent circuit with weighed input, defining pulse frequency. Right: Spike generation of the circuit.

While the accumulator clears, the comparator changes back to zero state. A small spike appears at the comparator comparable to an output spike of a nerve cell just in the minute, the accumulator is full.

If the input current loading the capacitor $K = C$ is i, the voltage over C grows with

$$\frac{du}{dt} = \frac{i}{C} \tag{13}$$

The fire frequency f for constant i is for $du = y_{max} = u_{max}$:

$$f = \frac{1}{dt} = \frac{i}{du\ C} = \frac{i}{u_{\max}\ C} \tag{14}$$

The brilliant idea behind nerve function is, to output the *strength of accumulation as a duration or pulse frequency f*. In neuro-computing, His idea [11] later comes back as ‘integrate and fire neuron’, compare for example [2]. Although his idea is over 100 years old, the specific association of spiking behavior with *interference integration* and the properties to *avoid overexposure* and to *code the intensity to pulse length* seems not to be well known.

Averaging Integration Methods

We found, interference integration incorporates the danger of divergence of the result. Looking for optimum integration methods, we find a class of “means”. Denoted by Wikipedia[1] very different means plays a technical rule. We would limit the choice to typical interference methods.

The principle of ‘mean’ reflects the integration with reset loop. Reset appears, if the value is maximal. The number of samples reflects the current.

By difference mean methods add a number of samples and divide the result by the number. For a linear ramp, the result is identical to the area integral.

Means used for Acoustic Cameras

The reconstruction process [4] adds first the delay-corrected channels k of microphones to get a single, averaging time function (bipolar) reconstruction $p_i(t)$. It represents the approximation of the original time function at the respective location i of the origin

$$p_i(t) = \frac{1}{n} \sum_{k=1}^{n} p_k(t + \tau_{ik}) \tag{15}$$

Integration in form of quadratic mean produces a single value $P_i(t)$ for each pixel (noise pressure as color) for a given time point t

$$P_i(t) = \sqrt{\frac{1}{n} \sum_{j=1}^{n} {p_{ij}}^2} \tag{16}$$

[1] http://en.wikipedia.org/wiki/Mean

The equation adds for a given time point t the squares of the next n samples producing a slot for each t. In practice, the t repeats at a distance of T.

A second mean $Q_i(t)$ allows a piecewise integration over some slots or frames to smoothen the acoustic film

$$Q_i(t) = \frac{1}{m} \sum_{k=1}^{m} P_{ik} \tag{17}$$

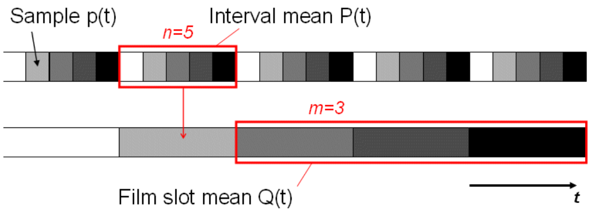

Fig. 9: Means used for Acoustic Camera

Using linear microphones and linear ADC's, the arithmetic mean in acoustics appears with the unit for sound pressure in Pascal (mostly in mPa). The sound level L is the logarithmic measure of the sound pressure related to a reference value, the threshold of human hearing $p_0 = 20\mu Pa$

$$L = 20 \log \left(\frac{P_i(t)}{p_0} \right) dB \tag{18}$$

For a continuous function the interference integral would appear with sample frequency f_s and sample time $T = n/f_s$ in the form

$$P_i(t) = \sqrt{\frac{1}{T} \int_0^T p_i^{\,2}(\tau)\, d\tau} \tag{19}$$

Conclusion

If a time function is leaving one-dimensional space, it becomes a wave. Main characteristic of a wave is that all points of the peak have the same delay to the source location.

Different, interaction aspects between type of conjunction, time function and type of integration influence interference integrals.

Defining the node-edge construct, calculations of interference conjunctions can work time- parallel. In vector notation, the output is a simple element-by-element conjunction of input vectors. The result of integration process is a number, that defines in images the lightness or color of the node or pixel.

Interferential functions correspond to relativity of wave expansion, not to gate location. Non-locality of interferential function reasons circuits, which deliver different behavior compared to state machines.

To highlight locations of wave interference, the specific integral calculus has to be non-linear (multiplication, exponation).

To process bipolar signals, sign switching restricts multiplication to two channels. Higher channel numbers need addition. Usability of multiplication for more then two signals restricts the time-functions to be monopolar (nerve).

Like acoustic time functions, photonic time function waves are symmetric. The conjunction process has additional type. Hence, acoustic and photonic processes need asymmetric integration. The mapping (image projection) needs any form of a non-linearity (quadratic mean; intensity, energy, quadrature).

Open integration produces integral divergence. To use the divergence technical, a time trigger has to stop the process.

Using a comparator resetting the integral value, the run-time characterizes the integral value. Analyzing the loop process, we find nerve pulse generation as a special case to code the integral value at the output of nerve cells in time-domain (frequency).

Using open integration with time limit, the quadratic mean technique of Acoustic Cameras appears as a specific integration method.

We could observe, that the interference network approach allows the calculation of systems in different fields (digital gates, light and acoustics). Pulse frequency coding of nerve cells shows a special case of interference integration.

Relating Works, References

1. Bullock, T.H.: Signals and signs in the nervous system: The dynamic anatomy of electrical activity. Proc. Natl. Acad. Sci. USA. 1997 January 7; 94(1): 1–6. http://www.ncbi.nlm.nih.gov/pmc/articles/PMC33648/
2. Gerstner, W., Kistler, W.M.: Spiking Neuron Models. Single Neurons, Populations, Plasticity. Cambridge University Press, 2002, paperback: ISBN 0 521 89079 9. http://icwww.epfl.ch/~gerstner/SPNM/node26.html#section-IF-nonlinear
3. Hamilton, W.R.: Theory of Systems of Rays. Transactions of the Royal Irish Academy, vol. 15 (1828), pp. 69{174.) Edited by David R. Wilkins 2001. http://www.maths.tcd.ie/~dwilkins/
4. Heinz, G.: Zur Physik bildgebender Rekonstruktion akustischer Bilder und Filme im Zeitbereich. 33. Deutsche Jahrestagung für Akustik, 2007, Uni Stuttgart, http://www.gfai.de/~heinz/publications/papers/2007_DAGA_Reko.pdf
5. Heinz, G.: Relativität elektrischer Impulsausbreitung.- Schlüssel zur Informatik biologischer Systeme. 39. IWK TU Ilmenau 27.-30.9.1994 S. 238-245, http://www.gfai.de/~heinz/publications/papers/1994_IWK.pdf

6. Heinz, G.: Introduction to Wave Interference Networks. Workshop 2010 'Autonomous Systems' 24.-29.10.2010 Camp de Mar, Mallorca, Spain, proceedings: Shaker-Verlag 2010, ISBN 978-3-8322-9514-1, ISSN 1866-7791 http://www.gfai.de/~heinz/publications/papers/2010_autosys.pdf
7. Heinz, G.: Interference Networks as a Generalizing Signal Theory. Plenary invited lecture. 7th Int. Conference on Computing and Information Technology IC2IT, May 11-12, 2011, King Mongkut's University of Technology, Bangkok, p. http://www.gfai.de/~heinz/publications/papers/2011_bangkok.pdf
8. Heinz, G.: Neuronale Interferenzen. Manuskript. 1993, 300 p. http://www.gfai.de/~heinz/publications/NI/index.htm
9. Heinz, G.: Interference integral animations (1995-2009). http://www.gfai.de/~heinz/publications/animations/index.htm
10. Heinz, G.: Interferential Projections in Inhomogeneous Space, web-page 1997 on http://www.gfai.de/~heinz/historic/distort/distort.htm
11. Lapicque, L.: Recherches quantitatives sur l'excitation électrique des nerfs traitée comme une polarisation. J. Physiol. Pathol. Gen. 9: (1907) 620–635
12. McCulloch, W., Pitts W.: A Logical Calculus of the Ideas Immanent in Nervous Activity. Bulletin of Mathematical Biophysics Vol 5, pp 115–133.
13. Packard, A.: Organization of cephalopod chromatophore systems: a neuromuscular image-generator. In: Abbott, N.J., Williamson, R., Maddock, L., *Cephalopod Neurobiology*, Oxford Press, 1995, pp. 331-367, http://www.gfai.de/~heinz/historic/biomodel/squids/squids.htm
14. Scilab 3.1.1: http://www.scilab.org/communities/developer_zone/scilab_versions